Lezioni 7 – 8
Disequazioni goniometriche.

IN AULA.

A) Elementari.

1. $\sin x > \dfrac{1}{2}$ $\qquad$ $S = \left\{ \dfrac{\pi}{6} + 2k\pi < x < \dfrac{5}{6}\pi + 2k\pi \right\}$

2. $\cos x > \dfrac{1}{2}$ $\qquad$ $S = \left\{ -\dfrac{\pi}{3} + 2k\pi < x < \dfrac{\pi}{3} + 2k\pi \right\}$

3. $\cos x < \dfrac{1}{2}$ $\qquad$ $S = \left\{ \dfrac{\pi}{3} + 2k\pi < x < \dfrac{5}{3}\pi + 2k\pi \right\}$

4. $|\sin x| > \dfrac{1}{2}$ $\qquad$ $S = \left\{ \dfrac{\pi}{6} + 2k\pi < x < \dfrac{5}{6}\pi + 2k\pi \right\} \cup \left\{ \dfrac{7}{6}\pi + 2k\pi < x < \dfrac{11}{6}\pi + 2k\pi \right\}$

5. $|\cos x| > \dfrac{\sqrt{2}}{2}$ $\qquad$ $S = \left\{ \dfrac{\pi}{4} + 2k\pi < x < \dfrac{3}{4}\pi + 2k\pi \right\} \cup \left\{ \dfrac{5}{4}\pi + 2k\pi < x < \dfrac{7}{4}\pi + 2k\pi \right\}$

6. $|\cos x| < \dfrac{\sqrt{2}}{2}$ $\qquad$ $S = \left\{ \dfrac{\pi}{4} + 2k\pi < x < \dfrac{3}{4}\pi + 2k\pi \right\}$

7. $\tan x \le \sqrt{3}$ $\qquad$ $S = \left\{ -\dfrac{\pi}{2} + k\pi < x < \dfrac{\pi}{3} + k\pi \right\}$

8. $3\tan x \le -\sqrt{3}$ $\qquad$ $S = \left\{ \dfrac{\pi}{2} + k\pi < x < \dfrac{5}{6}\pi + k\pi \right\}$

9. $|\tan x| > \dfrac{\sqrt{3}}{3}$ $\qquad$ $S = \left\{ \dfrac{\pi}{6} + k\pi < x < \dfrac{5}{6}\pi + k\pi \,,\, x \ne \dfrac{\pi}{2} + k\pi \right\}$

10. $2\cos^2 x < 1$ $\qquad$ $S = \left\{ \dfrac{\pi}{4} + k\pi < x < \dfrac{3}{4}\pi + k\pi \right\}$

11. $\cos^2 x \ge \dfrac{3}{4}$ $\qquad$ $S = \left\{ -\dfrac{\pi}{6} + k\pi \le x \le \dfrac{\pi}{6} + k\pi \right\}$

12. $\sin^2 x < \dfrac{1}{2}$ $\qquad$ $S = \left\{ -\dfrac{\pi}{4} + k\pi < x < \dfrac{\pi}{4} + k\pi \right\}$

13. $2\sin^2 x > 1$ $\qquad$ $S = \left\{ \dfrac{\pi}{4} + k\pi < x < \dfrac{3}{4}\pi + k\pi \right\}$

14. $3\,tg^2 x < 1$ $\qquad$ $S = \left\{ -\dfrac{\pi}{6} + k\pi < x < \dfrac{\pi}{6} + k\pi \right\}$

15. $0 \le \sin x \le \dfrac{\sqrt{3}}{2}$ $\qquad$ $S = \left\{ 2k\pi < x < \dfrac{\pi}{3} + 2k\pi \right\} \cup \left\{ \dfrac{2}{3}\pi + 2k\pi < x < \pi + 2k\pi \right\}$

B) Varie.

1. $2\cos^2 x + 3\cos x - 2 > 0$ $\qquad S = \left\{ 2k\pi \leq x < \dfrac{\pi}{3} + 2k\pi \right\} \cup \left\{ \dfrac{5}{3}\pi + 2k\pi < x \leq 2\pi + 2k\pi \right\}$

2. $\tan x (\tan x - 1) < 0$ $\qquad S = \left\{ k\pi < x < \dfrac{\pi}{4} + k\pi \right\}$

3. $2\cos^2 x \leq \sin x + 1$ $\qquad S = \left\{ \dfrac{\pi}{6} + 2k\pi \leq x \leq \dfrac{5}{6}\pi + 2k\pi \vee x = \dfrac{3}{2}\pi + 2k\pi \right\}$

4. $\sin x + \cos x < 0$ $\qquad S = \left\{ \dfrac{3}{4}\pi + 2k\pi < x < \dfrac{7}{4}\pi + 2k\pi \right\}$

5. $\cos x - \sin x < \sqrt{2}$ $\qquad S = \left\{ x \neq \dfrac{-\pi}{4} + 2k\pi \right\}$

6. $\sin x + 2\cos x > 1$ $\qquad S = \left\{ -2 \cdot \arctan(1/3) + 2k\pi < x < \pi/2 + 2k\pi \right\}$

7. $\sin x \cos x \leq \dfrac{1}{4}$ $\qquad S = \left\{ \dfrac{5}{12}\pi + k\pi \leq x \leq \dfrac{13}{12}\pi + k\pi \right\}$

8. $3\cos^2 x + \sin^2 x - 5\cos x + 1 \leq 0$ $\qquad S = \left\{ -\dfrac{\pi}{3} + 2k\pi \leq x \leq \dfrac{\pi}{3} + 2k\pi \right\}$

9. $\dfrac{1 + \cos^2 x}{1 + \sin x} > 2$ $\qquad S = \left\{ 2k\pi < x < \pi + 2k\pi \right\}$

10. $\sqrt{\sin x} > |\sin x|$ $\qquad S = \left\{ 2k\pi < x < \pi + 2k\pi , x \neq \dfrac{\pi}{2} + 2k\pi \right\}$

PROPOSTE lezioni 7-8.

Risolvere le seguenti equazioni e disequazioni.

1. $-\dfrac{1}{2}\le \cos x\le \dfrac{1}{2}$ $\qquad\qquad$ $S=\left\{-\dfrac{\pi}{6}+k\pi<x<\dfrac{\pi}{6}+k\pi\right\}$

2. $|2\cos x|>\sqrt{3}$ $\qquad\qquad$ $S=\left\{-\dfrac{\pi}{6}+k\pi<x<\dfrac{\pi}{6}+k\pi\right\}$

3. $2|\sin x|>\sqrt{3}$ $\qquad\qquad$ $S=\left\{\dfrac{\pi}{3}+k\pi<x<\dfrac{2}{3}\pi+k\pi\right\}$

4. $|\tan x|>1$ $\qquad\qquad$ $S=\left\{\dfrac{\pi}{4}+k\pi<x<\dfrac{3}{4}\pi+k\pi \wedge x\ne\dfrac{\pi}{2}+k\pi\right\}$

5. $2\sin^2 x-\sin x>0$ $\qquad\qquad$ $S=\left\{(2k+1)\pi<x<2(k+1)\vee\dfrac{\pi}{6}+2k\pi<x<\dfrac{5}{6}\pi+2k\pi\right\}$

6. $2\cos^2 x+3\cos x+1>0$ $\qquad$ $S=\left\{2k\pi\le x<\dfrac{2}{3}\pi+2k\pi\right\}\cup\left\{\dfrac{\pi}{3}+(2k+1)\pi<x\le2(k+1)\pi\right\}$

7. $\sqrt{3}\cos^2 x+3\cos x\sin x>0$ $\qquad\qquad$ $S=\left(-\dfrac{\pi}{6},\dfrac{\pi}{2}\right]\cup\left(\dfrac{5}{6}\pi,\dfrac{3}{2}\pi\right)$, periodo 2π

8. $|1+\cos x|<1-\sin x$ $\qquad\qquad$ $S=\left\{\dfrac{3}{4}\pi+2k\pi<x<\dfrac{7}{4}\pi+2k\pi\right\}$

9. $\ln(\sin x)+\ln(\cos x)+2\ln 2=0$ $\qquad\qquad$ $S=\{x=\pi/12+k\pi\vee x=5/12\pi+k\pi\}$

10. $2\tan x+\cos x-\dfrac{7}{4\cos x}>0$ $\qquad$ $S=\left\{\dfrac{\pi}{6}+2k\pi<x<\dfrac{\pi}{2}+2k\pi\right\}\cup\left\{\dfrac{5}{6}\pi+2k\pi<x<\dfrac{3}{2}\pi+2k\pi\right\}$

11. $\cos x-\sin x>0$ $\qquad\qquad$ $S=\left\{\dfrac{5}{4}\pi+2k\pi<x<\dfrac{9}{4}\pi+2k\pi\right\}$

12. $\sin x<\sqrt{3}(1-\cos x)$ $\qquad\qquad$ $S=\left\{\dfrac{\pi}{3}+2k\pi<x<2\pi(k+1)\right\}$

13. $\dfrac{|2\sin x+1|}{1-\sin x}\ge0$ $\qquad\qquad$ $S=\{x\ne\pi/2+2k\pi\}$

14. $2\tan x+\cos x-\dfrac{7}{4\cos x}>0$ $\qquad$ $S=\left\{\dfrac{\pi}{6}+2k\pi<x<\dfrac{\pi}{2}+2k\pi\right\}\cup\left\{\dfrac{5}{6}\pi+2k\pi<x<\dfrac{3}{2}\pi+2k\pi\right\}$

15. $\sin x+\cos x=\sqrt{2}$ $\qquad\qquad$ $S=\left\{x=\dfrac{\pi}{4}+2k\pi\right\}=\{x=2\cdot\arctan(\sqrt{2}-1)+2k\pi\}$

    nota : $\left(\tan\pi/8=\sqrt{2}-1\right)$

16. $\sin(x)+2\cos(x)>1$ $\qquad\qquad$ $S=\left\{x\in\mathbb{R}|\arctan\left(-\dfrac{3}{4}\right)<x<\dfrac{\pi}{2}, periodo=2\pi\right\}$

    o anche $\qquad\qquad\qquad$ $S=\left\{x\in\mathbb{R}|-2\cdot\arctan(\dfrac{1}{3})+2k\pi<x<\pi/2+2k\pi\right\}$